

Ideas for Connecting Inductive Program Synthesis and Bidirectionalization

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PEPM'12

A small “test”

Which function is this?

$$f_1 [a] = a$$

$$f_1 [a, b] = b$$

$$f_1 [a, b, c] = c$$

$$f_1 [a, b, c, d] = d$$

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And this one?

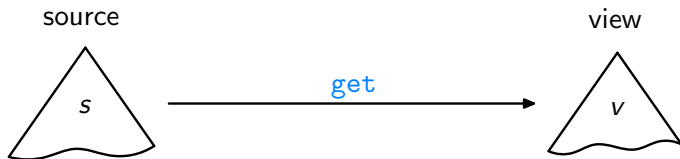
$$f_2 [] = []$$

$$f_2 [a] = [a]$$

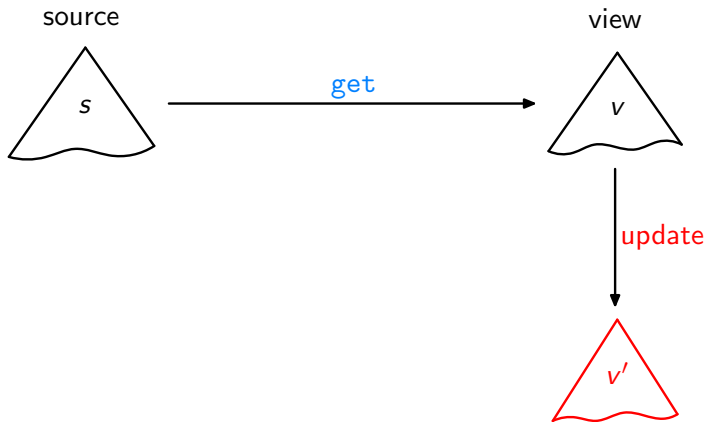
$$f_2 [a, b] = [b, a]$$

$$f_2 [a, b, c] = [c, b, a]$$

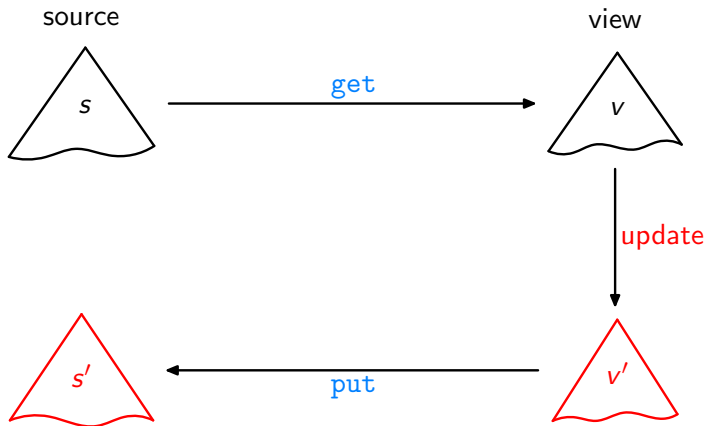
View-Update [Banc. & Sp., ACM TODS'81]



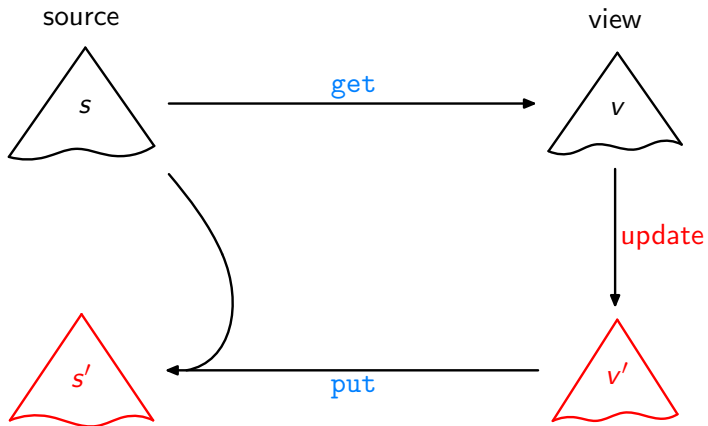
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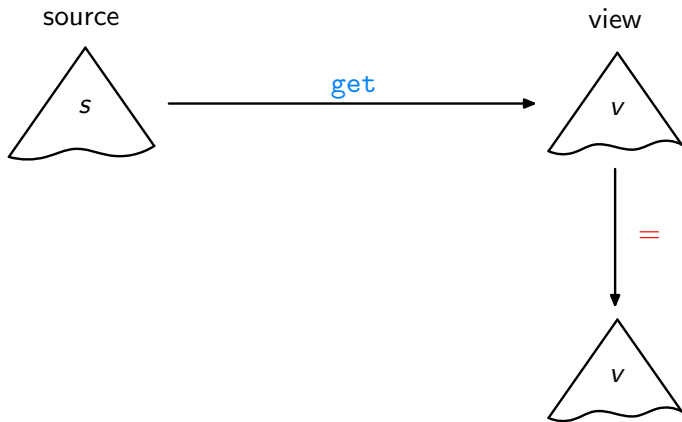
View-Update [Banc. & Sp., ACM TODS'81]



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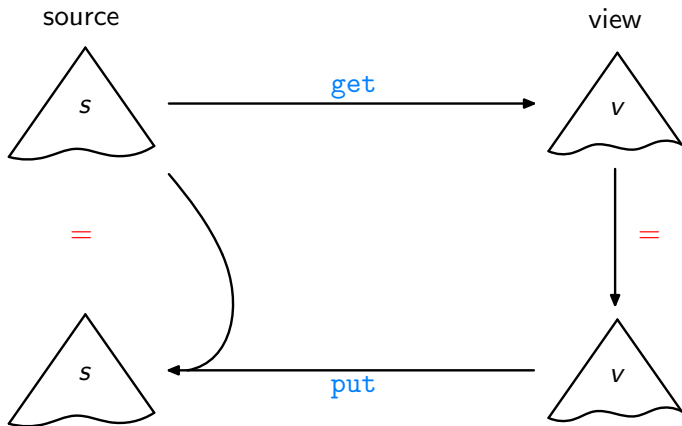


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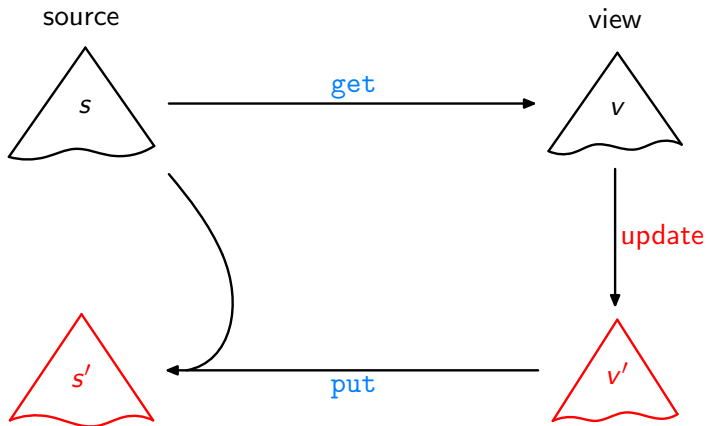
Acceptability / GetPut

View-Update [Banc. & Sp., ACM TODS'81]



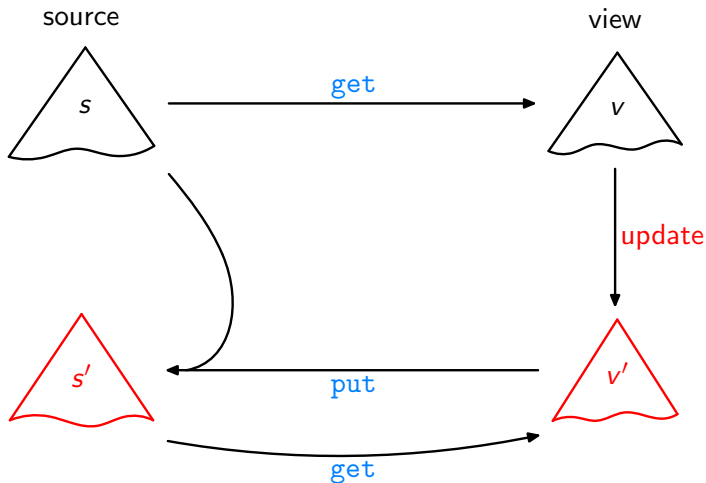
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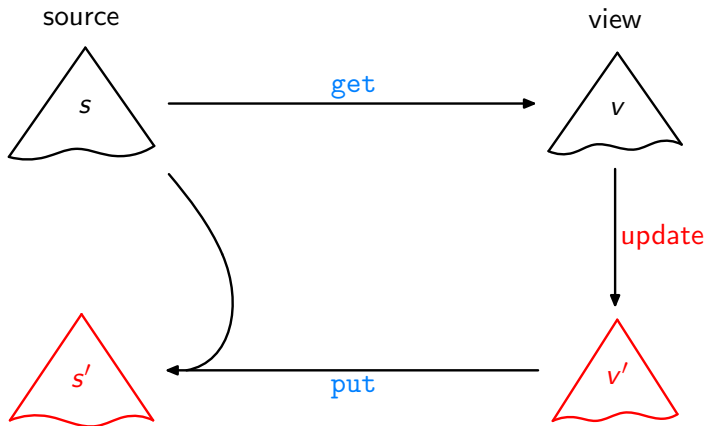
Consistency / PutGet

View-Update [Banc. & Sp., ACM TODS'81]

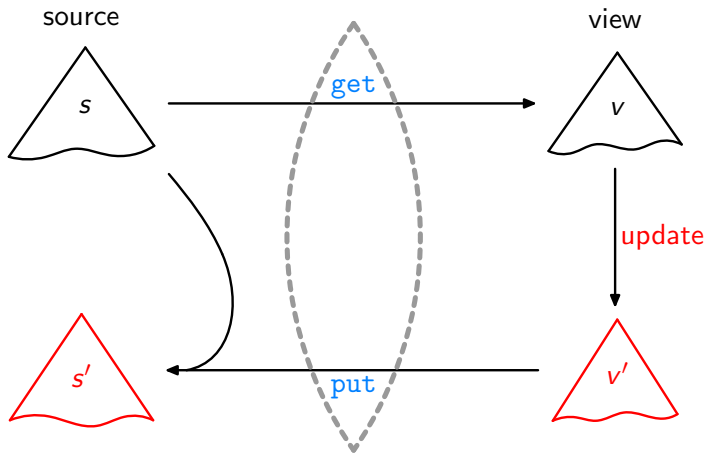


Consistency / PutGet

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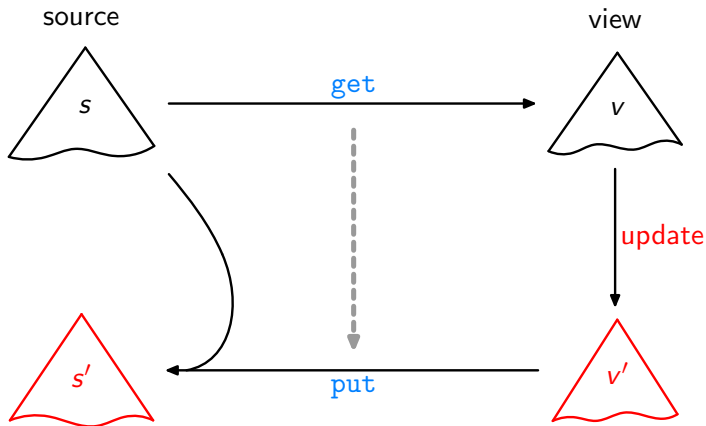
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Lenses, DSLs

[Foster et al., ACM TOPLAS'07, ...]

View-Update [Banc. & Sp., ACM TODS'81]



Bidirectionalization

[Matsuda et al., ICFP'07], [V., POPL'09], ...

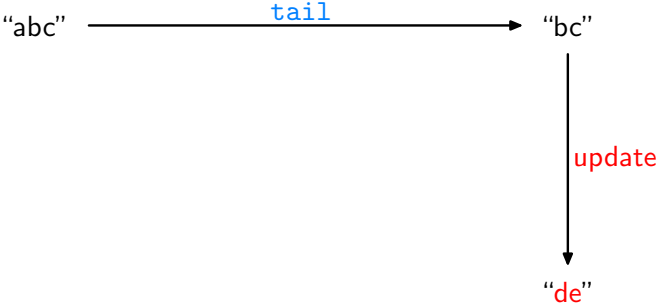
Bidirectionalization (BX)

Examples:

“abc” $\xrightarrow{\text{tail}}$ “bc”

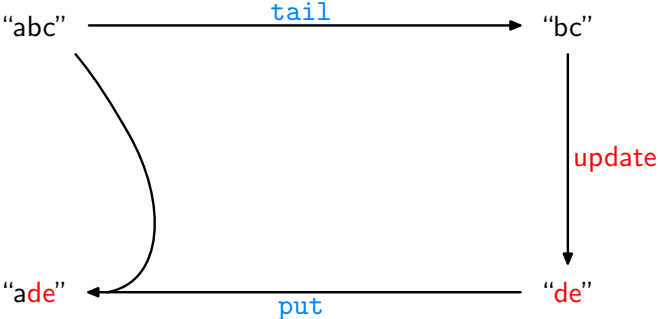
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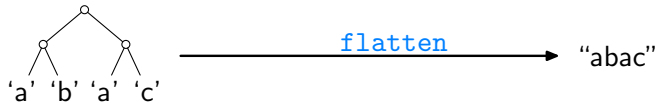
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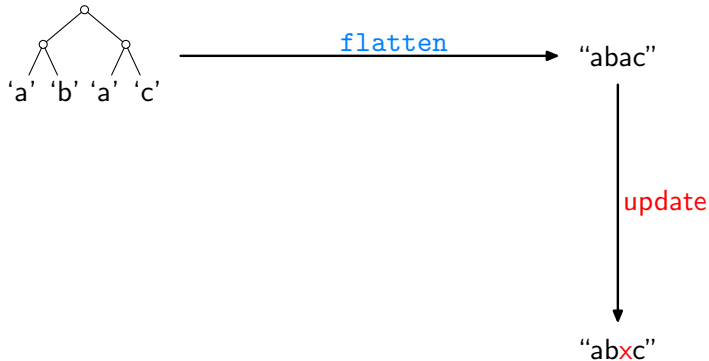
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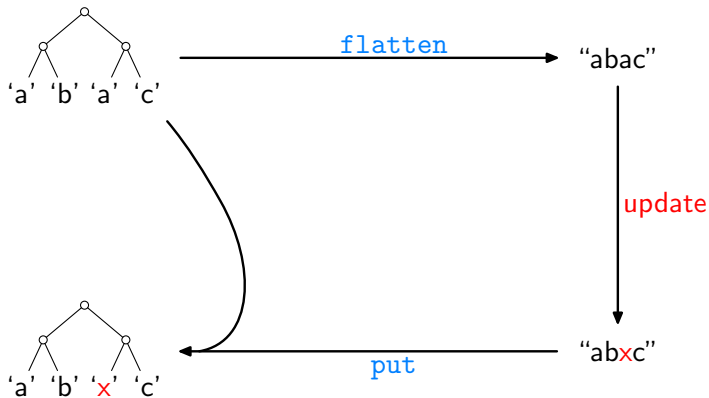
Bidirectionalization (BX)

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Examples:



Nondeterminism / Choices to make

Let `get = sieve` with:

<code>s</code>	<code>""</code>	<code>"a"</code>	<code>"ab"</code>	<code>"abc"</code>	<code>"abcd"</code>	<code>"abcde"</code>
<code>sieve s</code>	<code>""</code>	<code>""</code>	<code>"b"</code>	<code>"b"</code>	<code>"bd"</code>	<code>"bd"</code>

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Then, for example:

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put "abcd" "xy" = "axcy"
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or "axcyez" ?

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or "axcyez " ?

```
put "abcd" "xyz" = "axcy z"
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```
put "abcde" "xyz" = "axcyez"           or "axcyez " ?
```

```
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```
put "abcd" "x" = "axc"           or "ax" ? , or "cx" ?
```

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But that violates `put xs (get xs) = xs!`

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Better:

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put (x : xs) y | y == x    = (x : xs)
                | otherwise = [y]
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But that violates `put xs (get xs) = xs!`

Better:

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                | otherwise = [y]
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But “really intended”:

```
put (x : xs) y = (y : xs)
```

A slightly more complex case, with recursion

Let `get = init` with:

`init [x] = []`

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Possible, and correct:

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put xs ys | length ys == (length xs) - 1 = ys ++ [last xs]  
          | otherwise                    = ys ++ " "
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Problem: How to guide/control the possible choices?

Entry: Inductive Program Synthesis (IP)

Recall, I/O pairs for a function:

$$f_1 [a] = a$$

$$f_1 [a, b] = b$$

$$f_1 [a, b, c] = c$$

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From this, an IP system **automatically** generates the program:

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Or:

$$f_2 [] = []$$

$$f_2 [a] = [a]$$

$$f_2 [a, b] = [b, a]$$

$$f_2 [a, b, c] = [c, b, a]$$

Entry: Inductive Program Synthesis (IP)

Or:

$$\begin{aligned}f_2 [] &= [] \\f_2 [a] &= [a] \\f_2 [a, b] &= [b, a] \\f_2 [a, b, c] &= [c, b, a]\end{aligned}$$

Again automatically generated:

$$\begin{aligned}f_2 [] &= [] \\f_2 (x : xs) &= ((f_3 (x : xs)) : (f_2 (f_4 (x : xs)))) \\f_3 [x] &= x \\f_3 (x : xs) &= f_3 xs \\f_4 [x] &= [] \\f_4 (x : xs) &= (x : (f_4 xs))\end{aligned}$$

Entry: Inductive Program Synthesis (IP)

Or:

$$\begin{aligned}f_2 [] &= [] \\f_2 [a] &= [a] \\f_2 [a, b] &= [b, a] \\f_2 [a, b, c] &= [c, b, a]\end{aligned}$$

Or, through provision of `snoc` as “background knowledge”:

$$\begin{aligned}f_2 [] &= [] \\f_2 (x : xs) &= \text{snoc } (f_2 xs) x\end{aligned}$$

The master plan: BX + IP

Problem: Of the view-update laws

$$\text{put } xs \ (\text{get } xs) = xs$$

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only the first one directly delivers I/O pairs for `put`.

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$$\text{put } [a, b] \quad [a] \quad = [a, b]$$

$$\text{put } [a, b, c] \quad [a, b] \quad = [a, b, c]$$

$$\text{put } [a, b, c, d] \quad [a, b, c] \quad = [a, b, c, d]$$

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But then one would synthesize:

$$\text{put } xs \ ys = xs$$

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But then one would synthesize:

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1. possible solution: Enforce use of both arguments?

The master plan: BX + IP

First, on a simpler example, `get = head`:

```
put [a]          a = [a]  
put [a, b]       a = [a, b]  
put [a, b, c]    a = [a, b, c]  
put [a, b, c, d] a = [a, b, c, d]
```

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To avoid `put xs y = xs`, insist on use of `y`, i.e., something like:

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put xs y = (y : _)
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To avoid `put xs y = xs`, insist on use of `y`, i.e., something like:

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Starting from this hypothesis, practically only one reasonable path of synthesis, with result something like:

```
put xs y = (y : (tail xs))
```

The master plan: BX + IP

On the more complex example, `get = init`:

```
init [x]      = []  
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different “degrees” of use of `ys` in `put xs ys` are possible.

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put xs ys = (take (length ys) xs) ++ [last xs]
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Caution: Not every `put` generated (like) above automatically satisfies `get (put xs ys) = ys`.

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Or:

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Caution: Not every `put` generated (like) above automatically satisfies `get (put xs ys) = ys`. (But it's okay, trust IP.)

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2. possible solution: To after all generate I/O pairs for `put` from

$$\text{get} (\text{put } xs \ y) = y$$

as well, “inversion” of `get`.

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Then:

$$\text{put } xs \ y = \text{get}^{-1} \ y$$

as provider of further I/O pairs beside `put` `xs` (`get` `xs`) = `xs`.

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Then:

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as provider of further I/O pairs beside `put` `xs` (`get` `xs`) = `xs`.

Like, for `get` = `head`,

$$\text{head}^{-1} \ y = [y]$$

The master plan: BX + IP

2. possible solution: To after all generate I/O pairs for `put` from

$$\text{get} (\text{put } xs \ y) = y$$

as well, “inversion” of `get`.

Then:

$$\text{put } xs \ y = \text{get}^{-1} \ y$$

as provider of further I/O pairs beside `put` `xs` (`get` `xs`) = `xs`.

Like, for `get` = `head`,

$$\text{head}^{-1} \ y = [y]$$

or, better,

$$\text{head}^{-1} \ y = (y : -)$$

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or, better,

$$\text{head}^{-1} \ y = (y : _)$$

In this case, agrees with the other suggestion ...

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On the more complex example, `get = init`:

```
init [x]      = []  
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Use of

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init-1 ys = snoc ys _
```

to provide, beside:

```
put [a] [] = [a]  
put [a, b] [a] = [a, b]  
...
```

also:

```
put [a] [b] = [b, -]  
put [a, b] [] = [-]  
put [a, b] [c] = [c, -]  
...
```

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- ▶ Inductive Program Synthesis:
 - ▶ application of machine learning
 - ▶ detects/exploits regularities
 - ▶ hypothesis: captures programmer intentions
- ▶ Connection:
 - ▶ inductive program synthesis as a “helper”
 - ▶ either naively as a black box, or deeper integration
 - ▶ further ideas: I/O pairs per parametricity of `get`;
user impact through ad-hoc I/O pairs or
provision of background knowledge;
...

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